आंतरराष्ट्रीय बहुभाषिक शोध पत्रिका

प्रिंटिंग एरिया

Printing Area International Interdisciplinary Research Journal in Marathi, Hindi & English Languages

April 2018, Issue-45, Vol-01

Date of Publication
30 April 2018

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"Printed by: **Harshwardhan Publication Pvt.Ltd.** Published by **Ghodke Archana Rajendra** & Printed & published at Harshwardhan Publication Pvt.Ltd.,At.Post. Limbaganesh Dist,Beed -431122 (Maharashtra) and Editor **Dr. Gholap Bapu Ganpat.**

Reg.No.U74120 MH2013 PTC 251205
Harshwardhan Publication Pvt.Ltd.
At.Post.Limbaganesh,Tq.Dist.Beed
Pin-431126 (Maharashtra) Cell:07588057695,09850203295
harshwardhanpubli@gmail.com, vidyawarta@gmail.com
All Types Educational & Reference Book Publisher & Distributors / www.vidyawarta.com

Printing Area

Date of Publication
30 April 2018

Editorial Board & Advisory Committee

Govt. of India,
Trade Marks Registry
Regd. No. 3418002

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A NEW CLASS OF STARLIKE FUNCTION CORRESPONDING TO UNIFORMLY CONVEX FUNCTION

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ABSTRACT: The main object of this paper is to derive the sufficient conditions for the function $f(z)$ to be in the class $S_q^p(\gamma)$ which denotethe subclass of starlike functions corresponding to the family UC_p of uniformly convex functions.Further we investigate the distortion properties and radius of convexity estimates.Also, we determine extreme points and closure theorems.

AMS Subject Classification: 30C45
Key Words and Phrases: Convex Functions, Starlike Functions, Distortion Properties, Radius of Convexity Estimates, Extreme Points.

1. Introduction

Let A denote the class of functions $f(z)$ of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \qquad a_n \geq 0 , \qquad (1)$$

that are analytic in the open unit disk $U = \{ z : |z| < 1 \}$

Also, let T denote the class of functions $f(z)$ of the form

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n \qquad a_n \geq 0 ,$$

that are analytic in the open unit disk $U = \{ z : |z| < 1 \}$

A function $f(z) \in T$ is said to be starlike of order $\alpha, 0 \leq \alpha < 1$, if and only if

$$\mathrm{Re}\left(\frac{zf'(z)}{f(z)} \right) > \alpha$$

and convex if and only if

$$\mathrm{Re}\left(1 + \frac{zf''(z)}{f'(z)} \right) > \alpha$$

,with $z \in U$.

Definition (1) A function $f(z)$ is uniformly convex (uniformly starlike) in U if $f(z)$ is convex (starlike) and has the property that for every circular arc ς contained in U, with center ξ also in U ,the arc $f(\varsigma)$ is convex (starlike) with respect to $f(\xi)$. The class of uniformly convex functions is denoted by UC_p and the class of uniformly starlike functions by US_p [3] (also refer [4],[9]). These subclasses were introduced and defined by Goodman [1] and [2]. The class UC_p describes geometrically the domain of values of the expression $1+\frac{zf''(z)}{f'(z)}$, $z\in U$, $f(z)\in UC_p$ as a parabolic region [8].

$$\Re=\left\{\omega\in C:(\operatorname{Im}\omega)^2<2\operatorname{Re}\omega-1\right\}$$

Definition (2) [11] $f(z)\in UC_p$ if and only if

$$\operatorname{Re}\left(1+\frac{zf''(z)}{f'(z)}\right)\geq\left|\frac{zf''(z)}{f'(z)}\right|,z\in U \tag{3}$$

Definition (1.3) [3] For $-1\leq\gamma<1$, we let $S_q^p([\alpha_1],\gamma)$ denote the subclass of starlike functions corresponding to the family of uniformly convex function for $f(z)\in A$ s.t.

$$\operatorname{Re}\left\{\frac{z\left(H_q^p[\alpha_1,\beta_1]f(z)\right)'}{H_q^p[\alpha_1,\beta_1]f(z)}-\gamma\right\}\geq\left|\frac{z\left(H_q^p[\alpha_1,\beta_1]f(z)\right)'}{H_q^p[\alpha_1,\beta_1]f(z)}-1\right| \tag{4}$$

Where $H_q^p[\alpha_1,\beta_1]$ is called Dziok-Srivastava Operator[6].

Recently, Kanas and Srivastava[10], Srivastava and Mishra[4] and Vijaya and Murugusundaramoorthy[7] defined subclasses of uniformly convex and starlike function making use of various operators.

In this paper, for $f(z)\in T$ we suppose $G_q^p:T\to T$ be a linear operator defined by

$$G_q^p f(z)=z\,{}_p\Omega_q*f(z)$$

$$=z-\sum_{n=2}^{\infty}\frac{\prod_{j=1}^{q}\Gamma b_j}{\prod_{j=1}^{p}\Gamma a_j}\frac{\prod_{j=1}^{p}\Gamma\left(a_j+\alpha_j(n-1)\right)}{\prod_{j=1}^{q}\Gamma\left(b_j+\beta_j(n-1)\right)}a_n\frac{z^n}{(n-1)!}$$

$$G_q^p f(z)=z-\sum_{n=2}^{\infty}\Gamma\left(a_j,n\right)a_n z^n \tag{5}$$

Where

$$\Gamma\left(a_j,n\right)=\frac{\prod_{j=1}^{q}\Gamma b_j}{\prod_{j=1}^{p}\Gamma a_j}\frac{\prod_{j=1}^{p}\Gamma\left(a_j+\alpha_j(n-1)\right)}{\prod_{j=1}^{q}\Gamma\left(b_j+\beta_j(n-1)\right)}\frac{1}{(n-1)!} \tag{6}$$

and ${}_p\Omega_q=\frac{\prod_{j=1}^{q}\Gamma b_j}{\prod_{j=1}^{p}\Gamma a_j}\,{}_p\Psi_q$. Here ${}_p\Psi_q$ is the Fox-Wright function [5].

For $-1\leq\gamma<1$, we let $S_q^p(\gamma)$ denote the subclass of starlike functions corresponding to the family UC_p for $f(z)$ s.t.

$$\operatorname{Re}\left\{\frac{z\left(G_q^p f(z)\right)'}{G_q^p f(z)}-\gamma\right\} \geq \left|\frac{z\left(G_q^p f(z)\right)'}{G_q^p f(z)}-1\right| \qquad (7)$$

2. Main Results

Theorem 2.1 Let $f(z)$ be given by (2). If $-1\leq\gamma<1$ and $\sum_{n=2}^{\infty}(2n-1-\gamma)\Gamma(a_j,n)|a_n|\leq 1-\gamma$ then $f(z)\in S_q^p(\gamma)$ where $\Gamma(a_j,n)$ is given by (6).

Proof: By (7) for the class $S_q^p(\gamma)$, it suffices to show that

$$\operatorname{Re}\left\{\frac{z\left(G_q^p f(z)\right)'}{G_q^p f(z)}-\gamma\right\} \geq \left|\frac{z\left(G_q^p f(z)\right)'}{G_q^p f(z)}-1\right|$$

$$2\left|\frac{z\left(G_q^p f(z)\right)'}{G_q^p f(z)}-1\right|\leq 1-\gamma$$

or,

or,

$$2\left|\frac{z\left\{1-\sum_{n=2}^{\infty}\Gamma(a_j,n)a_n n z^{n-1}\right\}}{z-\sum_{n=2}^{\infty}\Gamma(a_j,n)a_n z^n}-1\right|\leq 1-\gamma$$

or,

$$2\left|\sum_{n=2}^{\infty}\Gamma(a_j,n)(1-n)a_n z^n\right|\leq(1-\gamma)\left|z-\sum_{n=2}^{\infty}\Gamma(a_j,n)a_n z^n\right|$$

or,

$$2\sum_{n=2}^{\infty}\Gamma(a_j,n)(n-1)|a_n|\leq(1-\gamma)\left\{1-\sum_{n=2}^{\infty}\Gamma(a_j,n)|a_n|\right\}$$

$$\sum_{n=2}^{\infty}(2n-1-\gamma)\Gamma(a_j,n)|a_n|\leq 1-\gamma$$

or,

Hence, the theorem is proved.

Distortion Properties and Radius of Convexity Estimates

Theorem 2.2 If $f(z)\in S_q^p(\gamma)$ then for $0<|z|=r<1;-1\leq\gamma<1$

$$r-\frac{1-\gamma}{(3-\gamma)\Gamma(a_j,2)}r^2\leq|f(z)|\leq r+\frac{1-\gamma}{(3-\gamma)\Gamma(a_j,2)}r^2$$

$$\because \sum_{n=2}^{\infty}a_n\leq\frac{1-\gamma}{(2n-1-\gamma)\Gamma(a_j,n)}$$

Proof:

$$0<|z|=r<1$$

Thus for

$$|f(z)|=\left|z-\sum_{n=2}^{\infty}a_n z^n\right|$$

$$\leq|z|+\left|\sum_{n=2}^{\infty}a_n z^n\right|\leq|z|+\sum_{n=2}^{\infty}|a_n||z^n|$$

$$\leq r+r^2\frac{1-\gamma}{(3-\gamma)\Gamma(a_j,2)};n=2$$

$$|f(z)|=\left|z-\sum_{n=2}^{\infty}a_n z^n\right|$$

And

$$\geq|z|-\left|\sum_{n=2}^{\infty}a_n z^n\right|\geq|z|-\sum_{n=2}^{\infty}|a_n||z^n|$$

Hence,

$$r-\frac{1-\gamma}{(3-\gamma)\Gamma(a_j,2)}r^2\leq|f(z)|\leq r+\frac{1-\gamma}{(3-\gamma)\Gamma(a_j,2)}r^2$$

and,

$$1-\frac{2(1-\gamma)}{(3-\gamma)\Gamma(a_j,2)}r \le \left|f^{'}(z)\right| \le 1+\frac{2(1-\gamma)}{(3-\gamma)\Gamma(a_j,2)}r$$

Extreme Points and Closure Theorem

Theorem 2.3 Let $f(z)$ be given by (2).

Define $f_1(z)=z$ and

$$f_n(z)=z-\frac{1-\gamma}{(2n-1-\gamma)\Gamma(a_j,n)}z^n, n\ge 2$$

Then $f(z)\in S_q^p(\gamma)$ if and only if $f(z)$ can be expressed as in the form

$$f(z)=\sum_{n=1}^{\infty}b_n f_n(z); b_n\ge 0, \sum_{n=1}^{\infty}b_n=1$$

$$f(z)=\sum_{n=1}^{\infty}b_n f_n(z)$$

Proof: If with

$$\sum_{n=1}^{\infty}b_n=b_1+\sum_{n=2}^{\infty}b_n=1$$

and $b_n\ge 0$ then

$$f(z)=b_1 f_1(z)+\sum_{n=2}^{\infty}b_n f_n(z)$$

$$=(1-\sum_{n=2}^{\infty}b_n)f_1(z)+\sum_{n=2}^{\infty}b_n\left[z-\frac{1-\gamma}{(2n-1-\gamma)\Gamma(a_j,n)}z^n\right]$$

$$f(z)=z-\sum_{n=2}^{\infty}\frac{1-\gamma}{(2n-1-\gamma)\Gamma(a_j,n)}b_n z^n$$

Now in view of Theorem 2.1

$$\sum_{n=2}^{\infty}(2n-1-\gamma)\Gamma(a_j,n)\left|\frac{(1-\gamma)}{(2n-1-\gamma)\Gamma(a_j,n)}b_n\right|=\sum_{n=2}^{\infty}b_n(1-$$

$$=(1-\gamma)(1-b_1)\le 1-\gamma$$

$$\{\because b_n\ge 0\}$$

$$f(z)\in S_q^p(\gamma)$$

Hence,

Conversely,

If $f(z)\in S_q^p(\gamma)$ then

$$a_n\le\frac{1-\gamma}{(2n-1-\gamma)\Gamma(a_j,n)}; n\ge 2$$

$$b_n=\frac{(2n-1-\gamma)\Gamma(a_j,n)}{1-\gamma}; n\ge 2$$

Setting and

$$b_1=1-\sum_{n=2}^{\infty}b_n$$

$$f(z)=\sum_{n=1}^{\infty}b_n f_n(z)$$

It follows that

And this completes the proof of the theorem.

Theorem 2.4 The class $S_q^p(\gamma)$ is closed under convex linear combination.

$$f^{(i)}(z)=z-\sum_{n=2}^{\infty}a_n^{(i)}z^n$$

Proof: Suppose ; $(i=1,2)$ are in $S_q^p(\gamma)$.

$$f(z)=(1-t)f^{(1)}(z)+tf^{(2)}(z)$$

Let ; $0\le t\le 1$

Then

$$f(z)=z-\sum_{n=2}^{\infty}\left[(1-t)a_n^{(1)}+ta_n^{(2)}\right]z^n$$

.

We have in view of Theorem 2.1

$$\sum_{n=2}^{\infty}(2n-1-\gamma)\Gamma(a_j,n)\left[a_n^{(1)}(1-t)+ta_n^{(2)}\right]$$

$$=(1-t)\sum_{n=2}^{\infty}(2n-1-\gamma)\Gamma(a_j,n)a_n^{(1)}+t\sum_{n=2}^{\infty}(2n-1-\gamma)\Gamma(a_j,n)a$$

$$\leq (1-t)(1-\gamma)+t(1-\gamma)=1-\gamma$$

Which shows that $f(z)\in S_q^p(\gamma)$.

REFERENCES

1) A.W.Goodman, On Uniformly Convex Functions, *Ann. polon Math.* 56(1991), 87-92.

2) A.W.Goodman, On Uniformly Starlike Functions, *J. Math. Anal. & Appl.*, 155(1991), 364- 370 .

3) Herb Silverman, A Class of Starlike Functions Defined by the Dziok Srivastava Operator, *Kyungpook Math. J.* 49 (2009), 95-106.

4) H.M.Srivastava and A.K.Mishra, Applications of Fractional Calculus to Parabolic Starlike and Uniformly Convex Functions, *Computer Math. Appl.*, 39 (2000), 57-69.

5) H.M.Srivastava and H.L.Manocha, A Treatise on Generating Functions, Halsted Press (Ellis Horwood Limited, Chichester), John Wiley and Sons, New York, Chichester, Brisbane and Toronto, 1984.

6) J.Dziok andH.M.Srivastava, Certain subclasses of Analytic Functions Associated With the Generalised Hypergeometric Function,*Integral Transform and Special Func.*,14(2003), 7-18.

7) K.Vijaya and G.Murugusundaramoorthy, Some Uniformly Starlike Functions with Varying Arguments, *Tamkang J.Math.*,35(1) (2004),23-28.

8) Oqlah Al-Refai and Maslina Darus, An Extension to the Owa-Srivastava Fractional Operator with Applications to Parabolic Starlike and Uniformly Convex Functions, *International Journal of Differential Equations*, Volume 2009, Article ID 597292, 18 Pages.

9) S.Kanas and F.Ronning, Uniformly Starlike and Convex Functions and other Related Classes of Univalent Functions, *Ann. Univ. Mariae Curie-Sklodowska Section A*, 53 (1999), 95-105.

10) S.Kanas and H.M.Srivastava, Linear Operators Associated with k-uniformly Convex Functions,*Integral Transform and Spec. Func.*,9(2000),121-132.

11) W.Ma and D.Minda, Uniformly Convex Functions, *Ann. polon. Math.*, 57 (1992),165-175.

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