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(M.A.Mar.& Pol.Sci.,B.Ed.Ph.D.NET.)

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45

A NEW CLASS OF STARLIKE FUNCTION CORRESPONDING TO UNIFORMLY CONVEX FUNCTION

I.B.BAPNA

Dept. of Mathematics
M. L. V. Govt. P. G. College, Bhilwara
Rajasthan, India

NIDHI JAIN

Assistant professor
K.V.S. University Gandhinagar, Gujrat

MAMTA SHARMA

Research Scholar,
M.L.V. Govt. College Bhilwara
Home address- 27, Paladi road, Subhash
Nagar Bhilwara

ABSTRACT: The main object of this paper is to derive the sufficient

conditions for the function $f(z)$ to be in

the class $S_q^p(\gamma)$ which denotes the subclass of starlike functions corresponding to the family UC_p

of uniformly convex functions. Further we investigate the distortion properties and radius of convexity estimates. Also, we determine extreme points and closure theorems.

AMS Subject Classification: 30C45

Key Words and Phrases: Convex Functions, Starlike Functions, Distortion Properties, Radius of Convexity Estimates, Extreme Points.

1. Introduction

Let A denote the class of functions $f(z)$ of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \quad a_n \geq 0$$

(1)

that are analytic in the open unit disk $U = \{z : |z| < 1\}$

Also, let T denote the class of functions $f(z)$ of the form

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n \quad a_n \geq 0$$

that are analytic in the open unit disk $U = \{z : |z| < 1\}$

A function $f(z) \in T$ is said to be starlike of order $\alpha, 0 \leq \alpha < 1$, if and only if

$$\operatorname{Re} \left(\frac{zf'(z)}{f(z)} \right) > \alpha$$

and convex if and only if

$$\operatorname{Re} \left(1 + \frac{zf''(z)}{f'(z)} \right) > \alpha \quad z \in U$$

with $f(z)$

Definition (1) A function $f(z)$ is uniformly convex (uniformly starlike) in U

if $f(z)$ is convex (starlike) and has the property that for every circular arc ξ contained in U , with center also in U , the arc $f(\xi)$ is convex (starlike) with

$f(\xi)$
 respect to UC_p . The class of uniformly
 convex functions is denoted by UC_p and
 the class of uniformly starlike functions
 US_p
 by [3] (also refer [4],[9]). These
 subclasses were introduced and defined
 UC_p
 by Goodman [1] and [2]. The class
 describes geometrically the domain of
 $1 + \frac{zf''(z)}{f'(z)}$, $z \in U$
 values of the expression
 $f(z) \in UC_p$
 as a parabolic region [8].

$$\Re = \left\{ \omega \in C : (\operatorname{Im} \omega)^2 < 2 \operatorname{Re} \omega - 1 \right\}$$

$f(z) \in UC_p$
Definition (2) [11] if and
 only if

$$\operatorname{Re} \left(1 + \frac{zf''(z)}{f'(z)} \right) \geq \left| \frac{zf''(z)}{f'(z)} \right|, z \in U \quad (3)$$

$-1 \leq \gamma < 1$
Definition (1.3) [3] For $-1 \leq \gamma < 1$, we let
 $S_q^p([\alpha_1], \gamma)$

denote the subclass of starlike
 functions corresponding to the family of

$$f(z) \in A \text{ uniformly convex function for } s.t. \operatorname{Re} \left\{ \frac{z(H_q^p[\alpha_1, \beta_1]f(z))'}{H_q^p[\alpha_1, \beta_1]f(z)} - \gamma \right\} \geq \left| \frac{z(H_q^p[\alpha_1, \beta_1]f(z))'}{H_q^p[\alpha_1, \beta_1]f(z)} - 1 \right| \quad (4)$$

$$H_q^p[\alpha_1, \beta_1]$$

Where is called Dziok-
 Srivastava Operator[6].
 Recently, Kanas and Srivastava[10],
 Srivastava and Mishra[4] and Vijaya and
 Murugusundaramoorthy[7] defined
 subclasses of uniformly convex and
 starlike function making use of various
 operators.

$$f(z) \in T$$

In this paper, for we suppose
 $G_q^p : T \rightarrow T$

be a linear operator defined by
 $G_q^p f(z) = z {}_p\Omega_q * f(z)$

$$= z - \sum_{n=2}^{\infty} \frac{\prod_{j=1}^q \Gamma b_j \prod_{j=1}^p \Gamma(a_j + \alpha_j(n-1))}{\prod_{j=1}^p \Gamma a_j \prod_{j=1}^q \Gamma(b_j + \beta_j(n-1))} a_n \frac{z^n}{(n-1)!}$$

$$G_q^p f(z) = z - \sum_{n=2}^{\infty} \Gamma(a_j, n) a_n z^n$$

(5)

Where

$$\Gamma(a_j, n) = \frac{\prod_{j=1}^q \Gamma b_j \prod_{j=1}^p \Gamma(a_j + \alpha_j(n-1))}{\prod_{j=1}^p \Gamma a_j \prod_{j=1}^q \Gamma(b_j + \beta_j(n-1))} \frac{1}{(n-1)!}$$

(6)

$${}_p\Omega_q = \frac{\prod_{j=1}^q \Gamma b_j}{\prod_{j=1}^p \Gamma a_j} {}_p\Psi_q$$

and Here ${}_p\Psi_q$ is the
 Fox-Wright function [5].

$$-1 \leq \gamma < 1 \quad S_q^p(\gamma)$$

For $-1 \leq \gamma < 1$, we let denote the
 subclass of starlike functions

corresponding to the family UC_p for $f(z)$
s.t.

$$\operatorname{Re} \left\{ \frac{z(G_q^p f(z))'}{G_q^p f(z)} - \gamma \right\} \geq \left| \frac{z(G_q^p f(z))'}{G_q^p f(z)} - 1 \right| \quad (7)$$

2. Main Results

Theorem 2.1 Let $f(z)$ be given by (2). If

$$-1 \leq \gamma < 1 \quad \sum_{n=2}^{\infty} (2n-1-\gamma) \Gamma(a_j, n) |a_n| \leq 1-\gamma$$

and

$$f(z) \in S_q^p(\gamma) \quad \Gamma(a_j, n)$$

then where is given by (6).

Proof: By (7) for the class $S_q^p(\gamma)$, it suffices to show that

$$\operatorname{Re} \left\{ \frac{z(G_q^p f(z))'}{G_q^p f(z)} - \gamma \right\} \geq \left| \frac{z(G_q^p f(z))'}{G_q^p f(z)} - 1 \right|$$

$$2 \left| \frac{z(G_q^p f(z))'}{G_q^p f(z)} - 1 \right| \leq 1-\gamma$$

or,

or,

$$2 \left| \frac{z \left\{ 1 - \sum_{n=2}^{\infty} \Gamma(a_j, n) a_n n z^{n-1} \right\}}{z - \sum_{n=2}^{\infty} \Gamma(a_j, n) a_n z^n} - 1 \right| \leq 1-\gamma$$

or,

$$2 \left| \sum_{n=2}^{\infty} \Gamma(a_j, n) (1-n) a_n z^n \right| \leq (1-\gamma) \left| z - \sum_{n=2}^{\infty} \Gamma(a_j, n) a_n z^n \right|$$

or,

$$2 \sum_{n=2}^{\infty} \Gamma(a_j, n) (n-1) |a_n| \leq (1-\gamma) \left\{ 1 - \sum_{n=2}^{\infty} \Gamma(a_j, n) |a_n| \right\}$$

$$\sum_{n=2}^{\infty} (2n-1-\gamma) \Gamma(a_j, n) |a_n| \leq 1-\gamma$$

or,

Hence, the theorem is proved.

Distortion Properties and Radius of Convexity Estimates

$f(z) \in S_q^p(\gamma)$

Theorem 2.2 If then for

$$0 < |z| = r < 1; -1 \leq \gamma < 1$$

$$r - \frac{1-\gamma}{(3-\gamma)\Gamma(a_j, 2)} r^2 \leq |f(z)| \leq r + \frac{1-\gamma}{(3-\gamma)\Gamma(a_j, 2)} r^2$$

$$Q \sum_{n=2}^{\infty} a_n \leq \frac{1-\gamma}{(2n-1-\gamma)\Gamma(a_j, n)}$$

Proof:

$$0 < |z| = r < 1$$

Thus for

$$|f(z)| = \left| z - \sum_{n=2}^{\infty} a_n z^n \right|$$

$$\leq |z| + \left| \sum_{n=2}^{\infty} a_n z^n \right| \leq |z| + \sum_{n=2}^{\infty} |a_n| |z^n|$$

$$\leq r + r^2 \frac{1-\gamma}{(3-\gamma)\Gamma(a_j, 2)}; n=2$$

$$|f(z)| = \left| z - \sum_{n=2}^{\infty} a_n z^n \right|$$

And

$$\geq |z| - \left| \sum_{n=2}^{\infty} a_n z^n \right| \geq |z| - \sum_{n=2}^{\infty} |a_n| |z^n|$$

Hence,

$$r - \frac{1-\gamma}{(3-\gamma)\Gamma(a_j, 2)} r^2 \leq |f(z)| \leq r + \frac{1-\gamma}{(3-\gamma)\Gamma(a_j, 2)} r^2$$

and,

$$1 - \frac{2(1-\gamma)}{(3-\gamma)\Gamma(a_j, 2)} r \leq |f'(z)| \leq 1 + \frac{2(1-\gamma)}{(3-\gamma)\Gamma(a_j, 2)} r$$

Extreme Points and Closure Theorem

$$f(z)$$

Theorem 2.3 Let be given by (2).

$$f_1(z) = z$$

Define and

$$f_n(z) = z - \frac{1-\gamma}{(2n-1-\gamma)\Gamma(a_j, n)} z^n, n \geq 2$$

$$f(z) \in S_q^p(\gamma)$$

Then if and only if can be expressed as in the form

$$f(z) = \sum_{n=1}^{\infty} b_n f_n(z); b_n \geq 0, \sum_{n=1}^{\infty} b_n = 1$$

$$f(z) = \sum_{n=1}^{\infty} b_n f_n(z)$$

Proof: If with

$$\sum_{n=1}^{\infty} b_n = b_1 + \sum_{n=2}^{\infty} b_n = 1$$

$$b_n \geq 0$$

and then

$$f(z) = b_1 f_1(z) + \sum_{n=2}^{\infty} b_n f_n(z)$$

$$= (1 - \sum_{n=2}^{\infty} b_n) f_1(z) + \sum_{n=2}^{\infty} b_n \left[z - \frac{1-\gamma}{(2n-1-\gamma)\Gamma(a_j, n)} z^n \right]$$

$$f(z) = z - \sum_{n=2}^{\infty} \frac{1-\gamma}{(2n-1-\gamma)\Gamma(a_j, n)} b_n z^n$$

Now in view of Theorem 2.1

$$\sum_{n=2}^{\infty} (2n-1-\gamma) \Gamma(a_j, n) \left| \frac{(1-\gamma)}{(2n-1-\gamma) \Gamma(a_j, n)} b_n \right| = \sum_{n=2}^{\infty} b_n (1-$$

$$= (1-\gamma)(1-b_1) \leq 1-\gamma$$

$$\{Q b_n \geq 0\}$$

$$f(z) \in S_q^p(\gamma)$$

Hence,

Conversely,

$$f(z) \in S_q^p(\gamma)$$

If then

$$a_n \leq \frac{1-\gamma}{(2n-1-\gamma)\Gamma(a_j, n)}; n \geq 2$$

$$b_n = \frac{(2n-1-\gamma)\Gamma(a_j, n)}{1-\gamma}; n \geq 2$$

Setting

and

$$b_1 = 1 - \sum_{n=2}^{\infty} b_n$$

$$f(z) = \sum_{n=1}^{\infty} b_n f_n(z)$$

It follows that

And this completes the proof of the theorem.

$$S_q^p(\gamma)$$

Theorem 2.4 The class is closed under convex linear combination.

$$f^{(i)}(z) = z - \sum_{n=2}^{\infty} a_n^{(i)} z^n$$

Proof: Suppose

$$(i=1, 2) \quad S_q^p(\gamma)$$

are in .

$$f(z) = (1-t)f^{(1)}(z) + tf^{(2)}(z)$$

Let

$$0 \leq t \leq 1$$

Then

$$f(z) = z - \sum_{n=2}^{\infty} [(1-t)a_n^{(1)} + ta_n^{(2)}] z^n$$

We have in view of Theorem 2.1

$$\sum_{n=2}^{\infty} (2n-1-\gamma) \Gamma(a_j, n) [a_n^{(1)}(1-t) + ta_n^{(2)}]$$

$$= (1-t) \sum_{n=2}^{\infty} (2n-1-\gamma) \Gamma(a_j, n) a_n^{(1)} + t \sum_{n=2}^{\infty} (2n-1-\gamma) \Gamma(a_j, n) a_n^{(2)}$$

$$\leq (1-t)(1-\gamma) + t(1-\gamma) = 1-\gamma$$

$$f(z) \in S_q^p(\gamma)$$

Which shows that

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