A FINITE HORIZON INVENTORY MODEL FOR DETERIORATING ITEMS WITH PROMOTIONAL EFFORT*

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Abstract: In this paper, we developed an inventory model for a finite time horizon in which needis dependent on promotional efforts. The major objective is to determine the optimal replenishment schedule such that total charges is reduced. It is determined that under these conditions, a solution of the model must exist and be unique. Next, algorithms are suggested to find the optimal solution, a numerical example is given to illustrate the model.

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1. Introduction

The contemporary climate, which is highly competitive, is fraught with various kinds of obstacles for businesses to meet challenging customers' demands while making a profit. People have employed promotion to promote their goods and services for many years now. More importantly, smaller companies and retailers exploited this to escalate sales and market share, while also pushing up their revenue. It is Freebies such as free goods and credit, as well as fair pricing cut to prices, are widely used in advertisements. Research, as well as promotional activities, is enhanced by this, consequently, and these two groups' attention is drawn to the importance of expanding these activities. Taylor is expanding into the middle of the forest in the same general direction. [1] When a sales goal is set, the quantity demanded is not just shifting but can expand and develop. Krishna et al. [2] Assisted the company in trying to examine the design of decentralized supply chains with the aid of retailers Tsao, Sheen [3] created a strategy for diminishing stock waste of dollars with dynamic pricing and marketing strategies that enable items to be restocked even after they have been paid for. 's Zhang et al. [4] committed to using finite horizons to optimizing the welfare by prioritizing three price/re-tender methods: stock market expansion, promotion, and marginal replenishment of stocks, has adopted a single object concept It's Grewal et al. [5] Concerning retail pricing and marketing, the field of three-based and targeting approaches, as well as the third-party merchandising, model and design, researchers have seen considerable progress. Sana's [6] The EOQ model has been put to the test, while needis influenced by salesmen's efforts and asset inventories are predictable, remain in tune. Sana [7] the price of the commodity determines the demands of the final customers as well as sales team efforts to extend. The Maiham and Karimi [8] Non-immediate declines in product needand occasional attempts at savings promotions can be simulated by using a model that has been specifically designed to simulate product prices and supply and demand-side fluctuations. Ca'rdenas-Barro'n, Sana [9] Gave the company new organizational ideas for a structure for two-tier supply chain: either making needbased on the efforts of sales teams or production team There have been many important studies that detail-related research projects done with the high-to-that kind that has applied for promotional activities in-needin inventory systems. (Palanivel and Uthayakumar) [10], Priyan et al. [11], and Taleizadeh [12]). Recently, Soni and Suthar [13] developed a non-price- and promotional sales loss distribution expansion algorithm for products that are not time-sensitive Researchers have discovered that there is a link between good results and a proper recovery plan and whether or no worsening of the deterioration.

The deteriorating of inventory systems can be thought of as a structural collapse. Factors that decrease the appeal, such as obsolescence, pilfering, evaporation, destruction, and lack of marginal utility influence cause inventory problems for several items. Many manufacturers face challenges in maintaining goods like food, chemicals, pharmaceuticals, hazardous chemicals, etc. over time. We must be careful when evaluating the charges of goods in calculating their future monetary value so that we include the time value of money in the calculation. Ghare and Schrader [14] First, a model of declining inventory was investigated. Aggarwal, Jaggi [15] Constructed an EOQ model to get the best order amount of decreasing goods in the shortest period. After that, all is fine; Chang et al. [16], Sana and Chaudhiri [17], and Ouyang et al. [18] for different inventory models of time-dependent demand, various types of deteriorating products were taken into account. Roy's [19] and Sana [20] For time-dependent demand, an EOQ model was investigated using various types of deteriorating products. Dye and Hsiesh [21] by considering the deterioration of inflation-dependent price-dependent products, an EOQ inventory model was created. Sarcar [22] the inventory model was tested for a time-varying deterioration rate with stock-dependent need while taking into account shortages and partial backlogging . Sony and Patel [23] established an inventory model for regular pricing and replenishment strategies, taking into consideration the time it takes for products to depreciate. Zhang et al. [24] by taking into considering a common rate and revival cycle decision-making issue developed an EOQ model for deterioration. Udayakumar and Geetha [25] Allowing for shortages and partial backlogs for decreasing price- and advertising-dependent products, an EOQ inventory model was presented.

The remainder of the paper is organized in the following manner. The notations and assumptions of the model are explained in Section 2. The third section is a mathematical model that has been developed. The theoretical findings are summarised in Section 4. The numerical results are shown in Section 5. Finally, Section 6” brings the paper to a close by suggesting alternative scientific directions.

2. Notations and Assumptions

2.1 Notations

Throughout this article, these notations are used:

n The no.of times [0, H] has been replenished (a decision variable)

t_i The ith replenishment (a decision variable), $i = 1, 2, ..., n$ with $0 \le t_1 < t_2 < ... < t_n < t_{n+1} = H$

s_i In ith replenishment period, time when the inventory level drops to 0 (a decision variable), $i = 1, 2, ..., n-1$ or n

H The extent to which a finite planning horizon exists

Q_i The number of orders in the ith replenishment period

A The charges of placing an order ($/order)

h The keeping charges ($/ unit time):

c The charges of acquisition ($/unit)

s The price of the shortage ($/per unit time)

θ The constant deterioration rate

ρ The promotional effort, $\rho \ge 1$

2.2 Assumptions

To build an inventory replenishment model, the following assumptions were used:

1. Throughout a finite planning horizon, a single object is considered.

2. The customer's needrate is given by the given expression.

   $D(\rho) = d_0 + \tau\left(1 - \frac{1}{1+\rho}\right)$, where d_0 is independent of the promotional effort and τ is a scale parameter that varies with the promotional effort [9]

3. The promotional effort charges in ith replenishment cycle PEC_i is a function of the promotional effort and the essential need that is growing. $PEC_i = k(\rho-1)^2\left[\int_{s_{i-1}}^{t_i} D(\rho)\,dt + \int_{t_i}^{s_i} e^{\theta(t-t_i)} D(\rho)\,dt\right]$, where $k > 0$. Both the market need and the promotional effort charges escalate as the promotional effort escalates.
4. The item deteriorates at a constant rate θ and there is no repair or replacement of the deteriorated units.
5. The shortfalls are allowed. The inventory model begins with shortage and finishes with zero inventories.

6. Lead duration is 0.

3. Model Formulation

The ith replenishment is complete at the time t_i. The amount received at the time t_i is used partially to meet the accumulated shortages in the prior cycle from time s_{i-1} to t_i $(s_{i-1} < t_i)$. The inventory t_i slowly reduces to zero s_i $(t_i < s_i)$. Since the inventory is exhausted by the collective effect of need and deterioration, the inventory level at time t during the ith replenishment cycle is governed by the following differential equation:

$$“\frac{dI(t)}{dt} = -D(\rho) - \theta I(t), \qquad t_i \le t \le s_i \quad .....(1) \tag{1}$$

With the boundary conditions $I(s_i) = 0$ solve the differential equations (1) we Have

$$I(t) = -\frac{D(\rho)}{\theta} + \frac{e^{\theta(s_i - t)} D(\rho)}{\theta}, \qquad t_i \le t \le s_i ” ....(2) \tag{2}$$

Now we need to figure out the various chargess of inventory, such as the charges of an order
placed during the ordering process. The ith replenishment cycle is

$$OC_i = A \quad ...(3) \tag{3}$$

During the holding period, the charges of holding ith replenishment cycle is given by

$$HC_i = h\int_{t_i}^{s_i} I(t)\,dt = \frac{hD(\rho)\left[\left\{(t_i - s_i)\theta - 1\right\} e^{-\theta s_i} + e^{-\theta t_i}\right] e^{\theta s_i}}{\theta^2} \quad ..(4) \tag{4}$$

The lack intensity at time t, $S(t)$, through the ith the following differential equation is used to describe the replenishment cycle:

$$“\frac{dS(t)}{dt} = -D(\rho), \qquad s_{i-1} \le t \le t_i, \quad i = 1, 2, ..., n ” ..(5) \tag{5}$$

With the boundary circumstance $S(s_{i-1}) = 0$ .solve the differential Eq. (5), we have

$$“S(t) = -D(\rho)(t - s_{i-1}), \qquad s_{i-1} \le t \le t_i, \quad i = 1, 2, ..., n ” ...(6) \tag{6}$$

The lack of charges during the ith replenishment phase is

$$SC_i = s\int_{s_i}^{t_{i+1}} -S(t)dt = \frac{sD(\rho)}{2}(t_i - s_{i-1})^2 \quad ..(7) \tag{7}$$

The order quantity in the ith replenishment cycle is

$$Q_i = \int_{s_{i-1}}^{t_i} D(\rho)\,dt + \int_{t_i}^{s_i} e^{\theta(t-t_i)} D(\rho)\,dt = D(\rho)(t_i - s_{i-1}) + \frac{D(\rho)\left[e^{\theta(s_i-t_i)} - 1\right]}{\theta} \qquad (8)$$

..(8)

The buying charges in the *i*th replenishment cycle is

$$PC_i = cQ_i = c\left[D(\rho)(t_i - s_{i-1}) + \frac{D(\rho)\left\{e^{\theta(s_i-t_i)} - 1\right\}}{\theta}\right] \qquad (9)$$

..(9)

The promotional effort charges in the *i*th replenishment sequence is

$$PEC_i = k(\rho-1)^2\left[\int_{s_{i-1}}^{t_i} D(\rho)\,dt + \int_{t_i}^{s_i} e^{\theta(t-t_i)} D(\rho)\,dt\right] = k(\rho-1)^2 D(\rho)\left[(t_i - s_{i-1}) + \frac{e^{\theta(s_i-t_i)} - 1}{\theta}\right]$$

(10)

As a result, the inventory system's total charges over the planning horizon H is as follows :

$$\begin{aligned} TC(n,\{s_i\},\{t_i\}) &= \sum_{i=1}^{n} OC_i + \sum_{i=1}^{n} HC_i + \sum_{i=1}^{n} SC_i + \sum_{i=1}^{n} PC_i + \sum_{i=1}^{n} PEC_i \\ &= nA + \sum_{i=1}^{n} \frac{hD(\rho)\left[\left\{(t_i - s_i)\theta - 1\right\}e^{-\theta s_i} + e^{-\theta t_i}\right]e^{\theta s_i}}{\theta^2} + \sum_{i=1}^{n} \frac{sD(\rho)}{2}(t_i - s_{i-1})^2 \\ &\sum_{i=1}^{n} c\left[D(\rho)(t_i - s_{i-1}) + \frac{D(\rho)\left\{e^{\theta(s_i-t_i)} - 1\right\}}{\theta}\right] + \sum_{i=1}^{n} k(\rho-1)^2 D(\rho)\left[(t_i - s_{i-1}) + \frac{e^{\theta(s_i-t_i)} - 1}{\theta}\right] \end{aligned}$$

(11)

here $s_0 = 0,\ t_1 > 0$ also $s_n < t_{n+1} = H$. The difficulty is now to determine $n, \{s_i\}, \{t_i\}$ such that $TC(n,\{s_i\},\{t_i\})$ in (11) is reduced .

4. Theoretical results and solution

For a permanent value of n, the essential conditions $TC(n,\{s_i\},\{t_i\})$ to be reduced are

$$\frac{\partial TC(n,\{s_i\},\{t_i\})}{\partial s_i} = 0, \quad i = 1,2,\ldots,n \text{ and } \frac{\partial TC(n,\{s_i\},\{t_i\})}{\partial t_i} = 0, \quad i = 1,2,\ldots,n$$

Which leads to

$$\frac{h\left[e^{\theta(s_i-t_i)} - 1\right]}{\theta} + c\left(e^{\theta(s_i-t_i)} - 1\right) + k(\rho-1)^2\left(e^{\theta(s_i-t_i)} - 1\right) = s(t_{i+1} - s_i) \quad (12)$$

And

$$\frac{h\left[e^{\theta(s_i-t_i)}-1\right]}{\theta}+c\left(e^{\theta(s_i-t_i)}-1\right)+k\left(\rho-1\right)^2\left(e^{\theta(s_i-t_i)}-1\right)=s\left(t_i-s_{i-1}\right)(13)$$

Respectively

Now we'll show that the conditions (12) and (13) are both necessary and sufficient for any Given n equations

To evaluate the absolute minimum $TC\left(n,\{s_i^*\},\{t_i^*\}\right)$. We know that the ideal value is 11 from 11. s_i Is the interior point between t_i and t_{i+1} because if $s_i = t_i$ or t_{i+1}, consequently, equation 12 is held. $TC\left(n,\{s_i\},\{t_i\}\right)$ is a continuous function that has been reduced to the smallest possible value over the compact set $\left[0,\mathrm{H}\right]^{2n}$ As a result, there exists a global minimum. The best value for is t_i cannot be on the border as $TC\left(n,\{s_i\},\{t_i\}\right)$ escalate when any one of t_i's shift to the endpoint 0 or H. In accumulation the solution to (12) and (13) as shown in theorem 1 below, is special. As a result, the required and proper conditions are equations (12) and (13) for the absolute minima " $TC\left(n,\{s_i^*\},\{t_i^*\}\right)$,,

Theorem 1: The solution to equations (12) and (13) not only exists but is also unique.

Proof: From (12) let

$$F\left(t_{i+1}\right)=s\left(t_{i+1}-s_i\right)-\frac{h\left[e^{\theta(s_i-t_i)}-1\right]}{\theta}-c\left(e^{\theta(s_i-t_i)}-1\right)-k\left(\rho-1\right)^2\left(e^{\theta(s_i-t_i)}-1\right)(14)$$

Taking the partial derivative of $F\left(t_{i+1}\right)$ concerning t_{i+1}, we obtain

$$F'\left(t_{i+1}\right)=s>0\ ..(15)$$

Consequently $F\left(t_{i+1}\right)$ is a strictly increasing function t_{i+1}, and hence $F\left(t_{i+1}\right)=0$ has a unique solution t_{i+1}. Similarly, by using (13) let

$$G\left(s_i\right)=\frac{h\left[e^{\theta(s_i-t_i)}-1\right]}{\theta}+c\left(e^{\theta(s_i-t_i)}-1\right)+k\left(\rho-1\right)^2\left(e^{\theta(s_i-t_i)}-1\right)-s\left(t_i-s_{i-1}\right)..(16)$$

Taking the partial derivative of $G\left(s_i\right)$ concerning s_i, we have

$$G'\left(s_i\right)=e^{\theta(s_i-t_i)}\left[h+\theta c+\theta k\left(\rho-1\right)^2\right]>0\ ..(17)$$

Consequently, given s_{i-1} and t_i, we know that (13) provides a unique solution for s_i.

This completes the proof.

Theorem 1 reduces the 2n-dimensional problem of finding $\{s_i^*\}$ and $\{t_i^*\}$ to a one-dimensional problem. Since $s_0^* = 0$ we need only to find t_1^* to generate s_1^* by (13), t_2^* by (12), and then the rest of $\{t_i^*\}$ and $\{s_i^*\}$ uniquely by repeatedly using (12) and (13). For any chosen t_1^*, if t_{n+1}^* = H then t_1^* is chosen correctly. Otherwise, we can easily find the optimal t_1^* by standard search schemes.

Algorithm 1 for Finding Optimal Schedule

Step 0 Choose two trivial values of t_1, Say $L = H/n$ and $U = 2H/n$. By using (13) and (12) to generate the rest of $\{s_i\}$ and $\{t_i\}$, and then compute the corresponding values of t_{n+1}, say M and V respectively.

Step 1 Let $t_1 = L + (U-L)(H-M)/(V-M)$ and calculate the corresponding t_{n+1}.

Step 2 if $t_{n+1} - H$ is sufficiently small, then $\{t_i^*\}$ and $\{s_i^*\}$ are obtained and stop.

Step 3 If $t_{n+1} - H$ is significantly large then zero, then set $U = t_1, \mathrm{V} = \mathrm{t}_{n+1}$ or set $L = t_1, \mathrm{M} = \mathrm{t}_{n+1}$ and go to

Step 1

For simplicity, let

$$TC(n) = TC\left(n, \{s_i^*\}, \{t_i^*\}\right) \quad ..(18)$$

Theorem 2: $TC(n)$ is convex in n.

Proof: See Appendix A.

The algorithm for determining the optimal number of replenishment n^* is summarized as follows:

Algorithm 2 for Finding Optimal Number and Schedule

Step 0 Choose two initial trial values of n^*, say n and n – 1. Use algorithm 1 to search for $\{t_i^*\}$ and $\{s_i^*\}$, and compute the corresponding $TC(n)$ and $TC(n-1)$, respectively

Step 1 If $TC(n) \geq TC(n-1)$, then compute $TC(n-2), TC(n-3), \ldots$, until we find $TC(k) < TC(k-1)$. Set $n^* = k$ and stop.

Step 2 If $TC(n) < TC(n-1)$, then compute $TC(n+1), TC(n+2), \ldots$, until we find $TC(k) < TC(k+1)$. Set $n^* = k$ and stop .

5. Numerical Example

Example 1: Consider the following data as an inventory condition to demonstrate the solution process: *A = $80*/unit, *c = $5*/unit, *s = $1.4*/unit/year, *h = $0.6*/unit/year, *k = 4,*

$\rho = 1.8$, $d_0 = 100$, $\tau = 50$, $\theta = 0.01$. Initially, we choose $n = 20$. By algorithm 2 we initiate a search n^* with n = 19 and 20. The search ends at n = 15. Because $TC(20) = 11576.40, TC(19) = 11495.60, TC(18) = 11414.71, \mathrm{TC}(17) = 11333.70$, $TC(16) = 11252.54, \mathrm{TC}(15) = 11171.22, TC(14) = 11189.68$, consequently the best number of replenishment $n^* = 15$. The results $\{t_i^*\}$ and $\{s_i^*\}$ values are specified in Table 1.

Table 1 Optimal solution of Example one

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
t_i^*	0.21	0.87	1.52	2.17	2.82	3.48	4.13	4.78	5.43	6.09	6.74	7.39	8.04	8.70	9.35
s_i^*	0.65	1.31	1.96	2.61	3.26	3.92	4.57	5.22	5.87	6.53	7.18	7.83	8.48	9.14	9.79

6. Conclusion

We developed an EOQ inventory model for finite time horizons in which needis influenced by promotional effort in this paper. We demonstrate that not only does the optimum replenishment schedule exist, but that it is also unique. Furthermore, we demonstrate that the overall charges of an inventory system is a convex function of the number of replenishments.

A variety of extensions to the proposed model are possible. For example, this model can be extended by considering the repair or replacement of deteriorated objects. Furthermore, by using promotional effort as a decision variable, the model can be expanded.

Appendix A: Proof of theorem 2

The proof is close to Friedman's [26], with the exception that shortages, depletion, and promotional effort are not taken into account. Let's say n orders are set in the range [0, H]. $TC(n, \{s_i\}, \{t_i\}) = TC(n, 0, H)$ The minimum value is defined by. Bellman's theory of optimality [27]. $TC(n, 0, H)$ is

$$TC^*(n, 0, H) = \underset{t \in [0, H]}{Minimize} \{TC^*(n-1, 0, \mathrm{t}) + TC(1, \mathrm{t}, H)\} \quad ..(19)$$

Let $t = H$ and thus $TC^*(n,0,H) < TC^*(n-1,0,H)$. The strict dissimilarity follows since the smallest in (19) occurs at an inner point. Thus $TC^*(n,0,H)$ is in n , the number is strictly decreasing. The following relationship emerges from the recursive application of (19):

$$t_i^*(n,0,\mathrm{h}) = t_i^*\left(n-j, \mathrm{t}_{n-j}^*(n,0,\mathrm{H})\right), \quad i = 1,2,\ldots,n-j-1 \text{ ..(20)}$$

To provide evidence that $TC^*(n,0,H)$ is strictly convex in n, we choose H_1 and H_2 such that

$$t_n^*(n+1,0,H_1) = t_{n+1}^*(n+2,0,H_2) = H \text{ ..(21)}$$

And $s_0^*(n+1,0,H_1) = s_0^*(n+2,0,H_2) = 0$. Employing the principle of optimality on (21) again, we have

$$TC^*(n+1,0,H_1) = \underset{t\in[0,H_1]}{Minimize}\left\{TC^*(n,0,\mathrm{t}) + TC(1,\mathrm{t},H_1)\right\} = TC^*(n,0,\mathrm{H}) + TC(1,\mathrm{H},H_1)$$

(15)

And

$$TC^*(n+2,0,H_2) = \underset{t\in[0,H_2]}{Minimize}\left\{TC^*(n+1,0,\mathrm{t}) + TC(1,\mathrm{t},H_2)\right\} = TC^*(n+1,0,\mathrm{H}) + TC(1,\mathrm{H},H_2)$$

(16)

Respectively Since H is an optimal interior point in both $(0,H_1)$ and $(0,H_2)$, we know that

$$\left.\frac{\partial TC^*(n,0,t)}{\partial t} + \frac{\partial TC(1,\mathrm{t},\mathrm{H}_1)}{\partial t}\right|_{t=H} = 0 \, (17)$$

And

$$\left.\frac{\partial TC^*(n+1,0,t)}{\partial t} + \frac{\partial TC(1,\mathrm{t},\mathrm{H}_2)}{\partial t}\right|_{t=H} = 0 \, (18)$$

Utilize the fact that

$$TC(1,a,b) = A + \frac{hD(\rho)\left[\left\{(a-v)\theta - 1\right\}e^{-\theta v} + e^{-\theta a}\right]e^{\theta v}}{\theta^2} + \frac{sD(\rho)}{2}(b-v)^2$$
$$+ c\left[D(\rho)(a-u) + \frac{D(\rho)\left\{e^{\theta(v-a)} - 1\right\}}{\theta}\right] + k(\rho-1)^2 D(\rho)\left[(a-u) + \frac{e^{\theta(v-a)} - 1}{\theta}\right]$$

(19)

Where u and v are the subsequent periods in the previous and current cycles when the inventory level drops to zero, respectively, we get

$$\left.\frac{\partial TC^*(n,0,t)}{\partial t}\right|_{t=H} = -\left.\frac{\partial TC(1,\mathrm{t},\mathrm{H}_1)}{\partial t}\right|_{t=H}$$

$$= \left(e^{\theta\left(s^*_{n+1}(n+1,0,H_1)-H\right)}-1\right)\left(\frac{h}{\theta}+c+k(\rho-1)^2\right)$$

$= s\left(H - s^*_n(n,0,H)\right)$ [By equation (13)] (20)

Here $s^*_n(n,0,H)$ and $s^*_{n+1}(n+1,0,H_1)$ when n orders are placed, is the corresponding last time the inventory amount reaches zero. In $[0,H]$ and $n+1$ orders are placed in $[0,H_1]$, respectively. Similarly ,

$$\left.\frac{\partial TC^*(n+1,0,t)}{\partial t}\right|_{t=H} = -\left.\frac{\partial TC(1,\mathrm{t},\mathrm{H}_2)}{\partial t}\right|_{t=H} = s\left(H - s^*_{n+1}(n,0,H)\right) \quad (21)$$

Subtracting (28) from (27), we have

$$\left.\frac{\partial}{\partial t}\left[TC^*(n,0,t)-TC^*(n+1,0,t)\right]\right|_{t=H} = s^*_{n+1}(n+1,0,H)-s^*_n(n,0,H)>0 \quad (22)$$

This implies that $TC^*(n,0,\mathrm{H})-TC^*(n+1,0,\mathrm{H})$ is a strictly increasing function of H. Thus,

$$TC^*(n,0,\mathrm{H})-TC^*(n+1,0,\mathrm{H}) < TC^*(n,0,\mathrm{H}_1)-TC^*(n+1,0,\mathrm{H}_1) \quad (23)$$

Again by (19) and the principle of optimality, we have

$$TC^*(n,0,\mathrm{H}_1)-TC^*(n+1,0,\mathrm{H}_1) = \underset{t\in[0,H_1]}{Minimize}\left\{TC^*(n-1,0,t)+TC(1,\mathrm{t},\mathrm{H}_1)-TC^*(n,0,\mathrm{H})-TC(1,\mathrm{H},\mathrm{H}_1)\right\}$$

(24)

Let $t=H$ in (35), we have

$$TC^*(n,0,\mathrm{H}_1)-TC^*(n+1,0,\mathrm{H}_1) < TC^*(n-1,0,\mathrm{H})-TC^*(n,0,\mathrm{H}) \quad (25)$$

By (30) and (32), we have

$$TC^*(n,0,\mathrm{H})-TC^*(n+1,0,\mathrm{H}) < TC^*(n-1,0,\mathrm{H})-TC^*(n,0,\mathrm{H}) \quad (26)$$

This implies $TC^*(n,0,H)$ is convex in n, and hence $TC(n)$ is also convex in n. Thus the theorem is

Proved

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